Computer Simulation of Dynamic Characteristics of Tandem Cold Rolling Process

Won Ho Lee* and Sang Ryong Lee**

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A computer simulation program that can analyze the dynamic behaviors of tandem cold rolling process without laborious experiments in actual mill was developed. By using this simulation program, the stability and accuracy of strip thickness control system were evaluated for various disturbance such as hot band gage, roll eccentricity, and deformation resistance of hot rolled strip. Herein the simulation program was described, and the results of simulation on feedback and feed-forward Automatic Gage Control were quoted as examples showing the effects of analysis on dynamic characteristics.

Key Words : Tandem Cold Rolling Process

Nomenclature

A, B, C, E : Coefficient matrix for variables
E_Y : Young's modulus
f : Forward slip
H, h : Entry and exit thickness
i : Suffix representing No. of stand
K : Mill structural stiffness
L : Distance between mill stands
P : Rolling force
R_W : Work roll radius
R_B : Back-up roll radius
S : Roll gap
S_R : Roll eccentricity
T_b, T_f : Backward and forward tension
T_S : Time constant for roll gap controller
T_V : Time constant for rolling speed controller
t : Time
U : Vector for input variables
V_e, V_o : Entry and exit speed of strip
V_R : Roll peripheral speed
W : Vector for disturbance variables
X : Vector for state variables
Y : Vector for output variables

* POSCO Technical Research Lab., Pohang-Shi, Kyungbuk, 790-785, KOREA, pc542764@posco. co. kr
** Dept. of Mech. Eng., K. N. U., Taegu-Shi, 702-701, KOREA, srlee@kyungpook. ac. kr

1. Introduction

In recent years, the quality requirements from the customers of cold rolled steel sheets have been steadily increasing in diversity and strictness. In particular, stringent gage accuracy is required for electrical steel, automobiles, and household appliance applications. To meet these quality requirements as well as to improve productivity, steel mills have equipped high performance Automatic Gage Control (AGC) system (Bryant, 1973; Tani, et al., 1988). The Automatic Gage Control system of the strip mill is one of the most advanced control systems in the steel industry. Various methods such as BISRA AGC, feed forward AGC, and feedback AGC have been proposed and applied for the control of the rolling phenomena in this field.

To improve the strip gage control system, main factors disturbing strip gage accuracy must be investigated and control system must be designed to prevent the disturbance (Gumi, et al., 1994; Sekiguchi, et al., 1992). The probable factors disturbing strip gage accuracy in steady state of cold rolling process are as follows: hot band gage variation, roll eccentricity, strip hardness variation, and the others.

The gage deviation due to the change in hardness corresponds to the skid mark of hot band steel strip (Edwards and Spooner, 1995).

Meanwhile, it is very difficult and time consuming to investigate the effects of above mentioned factors on strip gage accuracy in actual mill. Therefore, instead of actual mill, it is advantageous to develop the computer simulation tool, which can analyze the transient phenomenon of cold rolling process (Ogai, et al., 1991; Yoshida, et al., 1979). Moreover, by using the computer simulation of the dynamic characteristics of the rolling process, the effectiveness of gage control system can be examined in more detail and better design of the AGC systems can be achieved.

In this paper, the computer simulation program and the results of simulation on the transient phenomena of cold rolling process with actual mill data were described.

2. Mathematical Model of Cold Rolling Process

2.1 Thickness deviation and its control methods

The relationship between strip thickness and rolling force in flat rolling process is usually expressed by the following well known expression which is often referred to as gage meter equation.

$$h=\frac{P}{K}+S \tag{1}$$

The gage meter equation is graphically shown

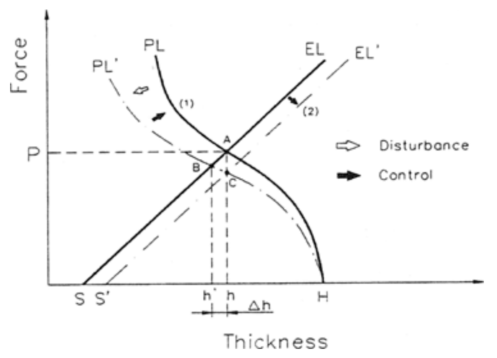


Fig. 1 Schematic diagram which represents gage meter equation.

in Fig. 1. The line EL in the figure is called as elastic line, which represents elastic behavior of mill housing system. In addition, the slope of EL represents mill structural stiffness. Meanwhile, the line PL is called as plastic line, which represents plastic behavior of rolled material.

Intersection of elastic line and plastic line, denoted by point A, determines the values of roll force P and exit thickness h. Therefore, aimed exit thickness h can be obtained by an accurate roll gap set at S. Because of unexpected causes, however, thickness deviation may exist in exit thickness. As shown in Fig. 1, if there is a disturbance that affects plastic line moving from PL downward to PL', exit thickness h' will deviate from aimed thickness by Δh.

To obtain the accurate thickness against external disturbance, two kinds of control methods can be considered. One is recovery of shifted plastic line PL' to initial plastic line PL (method (1) in Fig. 1): this is achieved by removing the causes of disturbance. Another is the roll gap control method: this can be attained by correcting the roll gap from S to S' (method (2) in Fig. 1). In this method, the amount of roll gap movement is determined to meet the aimed thickness in a distorted line of PL'. Since the roll gap control method is directive, it is widely used in gage control system.

2.2 Derivation of a dynamic model for tandem cold mill

In general, tandem cold mill is consisted of 4 to 5 stands, in which strip is rolled continuously from the first stand to the last one. It is a distinctive feature of tandem cold mill that there are interactions between adjacent stands. Figure 2 shows an outline of the rolling phenomena between stand i and $i+1$. In this figure, we can easily imagine that there are some interactions between the two contiguous stands.

By the inter-stand tension, every rolling phenomenon occurred in stand i affects stand $i+1$, and vice versa. Therefore, in order to understand the rolling phenomena more accurately, it is necessary to consider inter-stand tension when the dynamic rolling simulation is performed

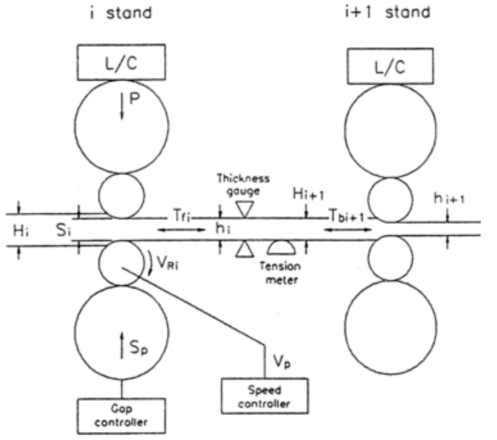


Fig. 2 Dynamic model of tandem cold rolling process.

(Yamamoto, et al., 1987; Nariharu, et al., 1979).

For the dynamic rolling simulation, the scheme of rolling process shown in Fig. 2 can be written by mathematical expression as following.

$$\boldsymbol{X}(t)=A\cdot\boldsymbol{X}(t)+B\cdot\boldsymbol{U}(t)+E\cdot\boldsymbol{W}(t) \quad (2)$$
$$\boldsymbol{Y}(t)=C\cdot\boldsymbol{X}(t) \quad (3)$$

where $\boldsymbol{X}=[\Delta h\ \Delta P\ \Delta V_R\ \Delta T_b]$,
$\boldsymbol{U}=[\Delta S_P\ \Delta V_P]$,
$\boldsymbol{W}=[\Delta H\ \Delta S_R]$,
$\boldsymbol{Y}=[\Delta h\ \Delta T_b]$

The first order differential equation of Eq. (2) is called the state equation. Moreover, algebraic equation of Eq. (3) is the output equation selected from the state variables. Therefore, dynamic characteristics of rolling process, which are time dependent phenomena such as transient changes in exit thickness and inter-stand tension, can be determined by solving Eq. (2) and Eq. (3) simultaneously by using integration method or Laplace transform technique.

To build up the state equation as in Eq. (2), several equations not only for rolling phenomena but also for dynamics of rolling mill control system are necessary. In this paper, we do not express the whole necessary equations precisely, but describe briefly the relationship between equations.

2.2.1 Modeling of rolling phenomena

(a) Gage meter equation

In actual mill, because there is often some eccentricity in back-up roll, the gage meter equation of Eq. (1) should be changed to take into account the roll eccentricity S_R. Thus, thickness of rolled strip is determined by following equation.

$$h_i=\frac{P_i}{K_i}+S_i+S_{Ri} \quad (4)$$

where P_i is rolling force which is expressed in a nonlinear form of the rolling parameters.

$$P_i=f(\cdots h_i,\ H_i,\ T_{bi},\ T_{fi}\cdots) \quad (5)$$

(b) Rolling speed

The exit speed V_O may be related to the roll peripheral speed V_R by using forward slip f_i. The forward slip is expressed as a function of rolling parameters under the condition such as mass flow passing through each vertical segment in the roll gap is constant.

$$V_{Oi}=(1+f_i)\cdot V_{Ri} \quad (6)$$
$$f_i=f(\cdots h_i,\ H_i,\ T_{bi},\ T_{fi}\cdots) \quad (7)$$

(c) Inter-stand tension

Because there is only one inter-stand tension between adjacent two stands, the forward tension of stand i equals to the backward one of stand $i+1$. The variation in backward tension of stand $i+1$, ΔT_{bi+1}, is occurred by elastic deformation between inter-stand, and this can be obtained by integration of the following equation.

$$\frac{dT_{bi+1}}{dt}=\frac{E_Y bh_i}{L}(V_{ei+1}-V_{Oi}) \quad (8)$$

The Eq. (8) means that variation of backward tension is proportional to the strip speed difference between entry side of stand $i+1$ and exit side of stand i.

2.2.2 Modeling of control system

Rolling mill is consisted of mechanical parts and electrical systems. Thus, it operates with time delay, which is called time constant between control signal and real action of actuator. The relationship between control signals and real actions can be written by their time constants as follows:

(a) Roll gap control system

$$\frac{dS_i}{dt}=\frac{1}{T_{si}}(-S_i+S_{pi}) \tag{9}$$

(b) Roll speed control system

$$\frac{dV_{Ri}}{dt}=\frac{1}{T_{vi}}(-V_{Ri}+V_{pi}) \tag{10}$$

2.2.3 State space model

Substituting Eq. (4) through Eq. (10) in the state equation Eq. (2) and arranging it, we can obtain the differential equations shown in Appendix for each state variables. State equations written in algebraic form can be expressed by matrix form for 5-stands tandem cold mill as follows:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\end{bmatrix}=\begin{bmatrix}A_1 & A_{12} & 0 & 0 & 0\\ A_{21} & A_2 & A_{23} & 0 & 0\\ 0 & A_{32} & A_3 & A_{34} & 0\\ 0 & 0 & A_{43} & A_4 & A_{45}\\ 0 & 0 & 0 & A_{54} & A_5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\end{bmatrix}+\begin{bmatrix}B_1 & 0 & 0 & 0 & 0\\ 0 & B_2 & 0 & 0 & 0\\ 0 & 0 & B_3 & 0 & 0\\ 0 & 0 & 0 & B_4 & 0\\ 0 & 0 & 0 & 0 & B_5\end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ u_3\\ u_4\\ u_5\end{bmatrix}$$
$$+\begin{bmatrix}E_1 & E_{12} & 0 & 0 & 0 & 0\\ 0 & E_2 & E_{23} & 0 & 0 & 0\\ 0 & 0 & E_3 & E_{34} & 0 & 0\\ 0 & 0 & 0 & E_4 & E_{45} & 0\\ 0 & 0 & 0 & 0 & E_5 & E_{56}\end{bmatrix}\begin{bmatrix}w_1\\ w_2\\ w_3\\ w_4\\ w_5\\ w_6\end{bmatrix}+\begin{bmatrix}E_1^d & 0 & 0 & 0 & 0\\ 0 & E_2^d & 0 & 0 & 0\\ 0 & 0 & E_3^d & 0 & 0\\ 0 & 0 & 0 & E_4^d & 0\\ 0 & 0 & 0 & 0 & E_5^d\end{bmatrix}\begin{bmatrix}\dot{w}_1\\ \dot{w}_2\\ \dot{w}_3\\ \dot{w}_4\\ \dot{w}_5\end{bmatrix} \tag{11}$$

where coefficient matrix A_{ij}, B_{ij}, and E_{ij} for each variable vector can be determined by rolling conditions and specifications of rolling mill. Therefore, the transient phenomena of rolling process can be obtained by integrating the Eq. (11) with numerical analysis technique.

3. Conditions of Simulation

Using the equations explained above, the dynamic simulation program was developed.

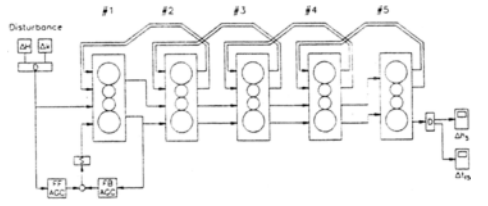


Fig. 3 Structure of dynamic rolling simulation program.

Figure 3 illustrates the overall structure of simulation program that was developed by Simulink (MathWorks, Inc., 1993). Since the program is developed by graphic processing tool, which is called Graphic User Interface (GUI) program, it is very convenient to correct the input data and to check the results during the simulation. By using this simulation program, the dynamic characteristics of cold rolling process was analyzed and accuracy of thickness control system was evaluated as a function of disturbance.

As an ideal case, if there were no disturbance in steady state rolling process, thickness deviation as well as perturbation of inter-stand tension would not occur. In real case, however, disturbance can not be neglected. In this work, disturbance is approximated by a time dependent sinusoidal wave as follows:

(a) Thickness variation in hot band gage

$$\Delta H_1=\sum C_i\cdot\sin(2\pi\cdot f_i\cdot t) \tag{12}$$

where C_i, f_i=amplitude and frequency of thickness deviation in hot rolled strip.

(b) Backup roll eccentricity

The roll gap change due to the backup roll eccentricity varies with the frequency corresponding to revolution of the rolls as follows:

$$\Delta S_R=C_R\cdot\sin(2\pi\cdot f_R\cdot t) \tag{13}$$

$$f_R=\frac{1}{2\pi}\frac{V_R}{R_B}\left(\frac{1000}{60}\right) \tag{14}$$

where C_R, f_R=amplitude and frequency of roll eccentricity.

(c) hardness deviation

$$\Delta k=C_k\cdot\sin(2\pi\cdot f_k\cdot t) \tag{15}$$

where C_k, f_k=amplitude and frequency of longitudinal hardness deviation.

Table 1 Specifications of tandem cold rolling mill.

Items \ Stand No.		1	2	3	4	5
Radius of work roll (mm)		267	264	270	276	184
Radius of back-up roll (mm)		708	665	685	705	650
Mill structural stiffness (MN/mm)		4512	4512	4512	4512	4512
Distance between stands (mm)		4600	4600	4600 4600	4600	
Time constant (sec)	Roll gap controller	0.05	0.05	0.05	0.05	0.05
	Rolling speed controller	0.1	0.1	0. 1	0.1	0.1

Table 2 Rolling conditions used in simulation.

Items \ Stand No.	1	2	3	4	5
Entry thickness (mm)	2.3	1.633	0.988	0.658	0.511
Exit thickness (mm)	1.633	0.988	0.658	0.511	0.355
Reduction ratio (%)	29.0	39.0	33.0	22.0	30. 0
Strip width (mm)	808	808	808	808	808
Entry unit tension (N/mm^2)	82.6	123.8	120	90.2	93.1
Exit unit tension (N/mm^2)	123.8	120	90.2	93.1	56.6
Rolling speed (m/s)	3.73	6.17	9.27	11.93	17.17

Table 1 shows the specification of rolling mill. In addition, Table 2 shows rolling conditions used in this simulation. The data in tables were collected from an actual 5-stands tandem cold mill of POSCO.

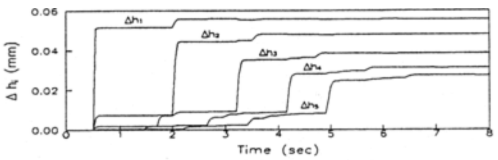
4. Simulation Results and Discussion

4.1 Dynamic characteristics of tandem cold rolling process

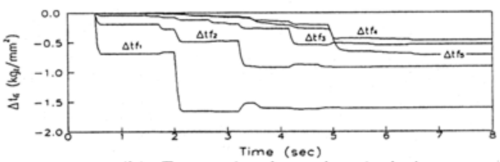
The rolling disturbance changes periodically with a certain amount of amplitude as mentioned in previous section. The trends of rolling parameters observed at the delivery side of mill will become sinusoidal wave when simulation is conducted under the assumption of sinusoidal wave type of disturbance. In that case, it is very difficult to determine accurately the trend of outputs with respect to that of inputs. In this work, therefore, step function type of disturbance with constant amplitude was used for the understanding of dynamic characteristics of tandem cold rolling process.

Figure 4 shows the changes in rolling parameters for each stand by the step type of disturbance in hot band gage. Thickness deviation of 0. 115mm, which was equivalent to 5% of base thickness of 2.3mm, was assumed. Figure 4 (a) represents the delivery side thickness deviation (Δh_i) of each stand with respect to the disturbance of hot band gage ($+\Delta H_1$). However, absolute value of the deviation was reduced gradually as rolling proceeded. Backward unit tension deviation (Δt_{bi}) of each stand was decreased to negative direction when the disturbance applied while the deviation was getting smaller as the rolling proceeded.

Based on the computer simulation using step type of disturbance, the characteristics of tandem cold rolling process were summarized follows:

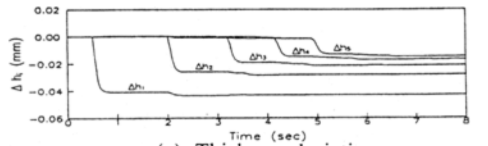


(a) Thickness deviation

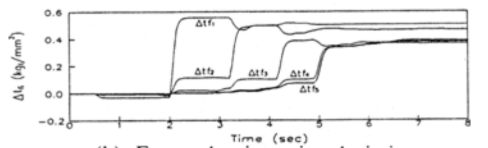


(b) Forward unit tension deviation

Fig. 4 Variation of gages and forward unit tensions by the step change of hot band gage.

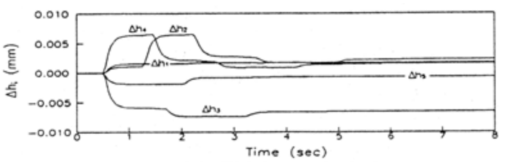


(a) Thickness deviation

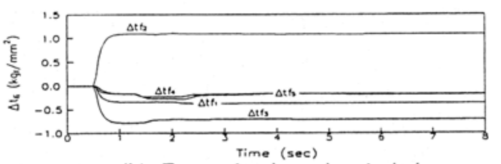


(b) Forward unit tension deviation

Fig. 5 Variation of gages and forward unit tensions by the step decrease of the roll gap of No. 1 stand.



(a) Thickness deviation



(b) Forward unit tension deviation

Fig. 6 Variation of gages and forward unit tensions by the step increase of the rolling speed of No. 3 stand.

1) Thickness deviation caused by the increase of entry thickness was naturally damped to a certain degree as the rolling proceeded under no extra thickness control.

2) An increase of hot band thickness reduced the unit tension of all stands.

3) Thickness deviation occurred abruptly when the starting point of disturbance arrives at relevant stand and increased small amount when the forward unit tension changed.

4) Thickness deviation occurred at rear stand influenced to next stand with a time delay, but tension deviation influenced to adjacent stands immediately.

4.2 Change of rolling characteristics by thickness control methods

In order to correct thickness deviation caused by disturbance during rolling, automatic gage control system is used by means of roll gap and rolling speed control. Therefore, it is very important to understand control concept of using these means in terms of rolling phenomena.

4.2.1 Effects of roll gap movement

Figure 5 shows the changes in delivery side thickness of each stand and in backward unit tension when roll gap of stand No. 1 increased from steady state by 0.082mm which was equivalent to 5% of delivery thickness, 1.633mm. As the roll gap of stand No. 1 was increased, thickness deviation (Δh_i) was changed instantly but saturated to a constant value after a while. As shown in Figure 5 (b), the thickness deviations caused by roll gap increase at stand No. 1 were reduced gradually as rolling proceeded. However, the backward unit tensions of all stands were increased abruptly and decreased gradually with the time.

4.2.2 Effects of rolling speed change

Figure 6 shows the effects of rolling speed on rolling phenomena with increasing the speed of stand No. 3 by 1% of initial speed, 9.3m/s. As increasing the speed of stand No. 3, thickness of the stand was being decreased but those of other stands were disturbed only when speed changed and returned to original status after a while. In case of backward tension, the speed increase resulted in increasing backward tension of stand No. 3 but in decreasing that of stand No. 4. As the rolling speed increases, in general, strip speed of delivery side increases and that of entry side decreases. Correspondingly delivery strip tension decreases and entry tension increases. Since strip tension influences on strip thickness, proper rolling speed control is necessary for an accurate strip thickness.

4.3 Strip thickness deviation caused by sinusoidal disturbance

The actual disturbance during rolling process appeared not as a step function but as a cyclic function that was time dependent. Therefore, to study the effects of the aforementioned disturbance on change of rolling parameters, the simulation of transient characteristics with sinusoidal disturbance should be carried out. The magnitude and frequency of disturbance are given in Table 3, which were obtained from the FFT (Fast Fourier Transform) analysis of the measured disturbance data in actual mill.

Figure 7 shows the thickness deviation at the delivery side of each stand when all disturbance parameters were influenced simultaneously. It was shown that the thickness deviation was getting reduced as rolling processed. Residual thickness deviation (Δh_5) of final product was found to be 15.4 μm.

In case of wavy mode, the frequency of thickness deviation was very similar in mode to that of disturbance although the amplitudes of high and low frequency components reduced significantly. According to the simulation results for individual

Table 3 Amplitudes and frequencies used as a disturbance for tandem cold rolling simulation.

Disturbance		Amplitude	Frequency (Hz)
Hot band gage deviation, ΔH_1	High frequency	0.02 mm	1
	Low frequency	0.05 mm	0.1
BUR eccentricity, ΔS_R		0.01 mm	varies with speed
Hardness deviation, Δk		21.1 N/mm^2	0.033

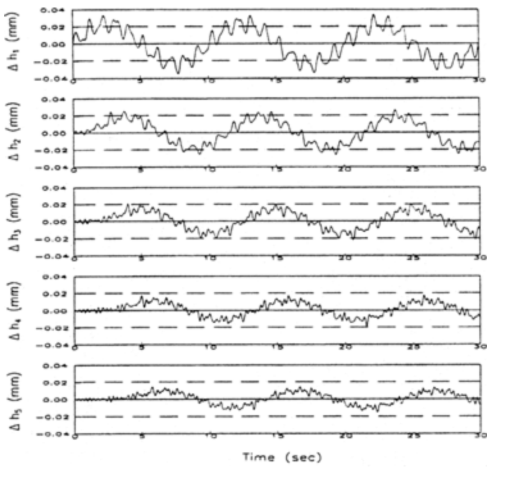


Fig. 7 Thickness deviation at delivery side of each stand by the sinusoidal disturbance.

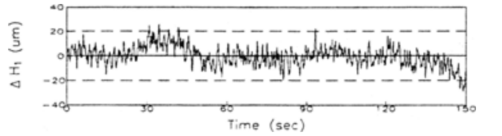
parameter, the most influential parameter of disturbance that created thickness deviation was the initial thickness deviation of the hot rolled strip and the least one was the hardness deviation of hot rolled strip.

4.4 Effects of feed forward and feed back AGC on strip thickness accuracy

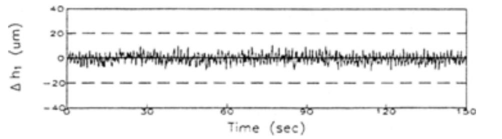
Feed forward AGC (FF-AGC) of stand No. 1 is a thickness controller to control aimed thickness with the known thickness deviation of supplied hot band at the entry side of stand No. 1. Feedback AGC (FB-AGC) is a controller to correct thickness deviation which might be generated at the delivery side of stand No. 1. In this work, the effects of thickness control by FF-AGC and FB-AGC were examined by using the developed simulation program.

Figure 8 shows charts of thickness deviation measured at the entry and delivery sides of stand No. 1 of an actual cold rolling mill. Thickness deviation of hot band gage, Fig. 8 (a), was used as the disturbance in computer simulation.

Figure 9 illustrates the effect of using AGC on thickness deviation. Figure 9 (a) shows the calculated thickness deviation at delivery side of stand No. 1 when both of FF-AGC and FB-AGC were used. It showed that the thickness deviation was reduced dramatically by applying the AGCs, comparing to Fig. 9 (b), in which the gage con-

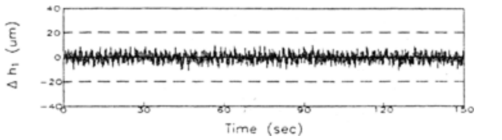


(a) Entry side thickness deviation

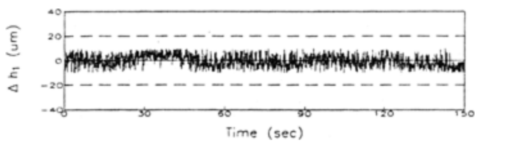


(b) Delivry side thickness deviation

Fig. 8 Measured thickness deviation at entry and delivery side of No. 1 stand.



(a) Entry side thickness deviation (AGC on)



(b) Entry side thickness deviation (AGC off)

Fig. 9 Calculated thickness deviation at delivery side of No. 1 stand.

troller was not used. In addition, it is noted that the calculated thickness deviation shown in Fig. 9 (a) coincides very well with the measured one of Fig. 8 (b). This shows that the simulation tool can be used effectively as an off-line predictor of thickness deviation.

5. Conclusion

In this work, a simulation program was developed to analyze the transient rolling characteristics on the basis of general cold rolling theories and dynamics of rolling mill. The simulation results with actual mill data showed that the developed simulator has enough accuracy in calculation of dynamic characteristics of tandem cold rolling process. Therefore, the developed simulator could be effectively used to calculate the amount of thickness deviation and to study the function of thickness controller used for reducing thickness deviation.

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Appendix: Algebraic derivation of state equation

In this appendix, a set of state equation formed as the first order differential equation is suggested as follows:

$$\frac{dx_{i,1}}{dt}=\left[\frac{\partial\Delta P_{Ai}}{\partial\Delta h_i}\frac{1}{T_{pi}K_i}-\frac{1}{T_{si}},\ \frac{-1}{T_{pi}K_i}+\frac{1}{T_{si}K_i},\ 0,\ \frac{\partial\Delta P_{Ai}}{\partial\Delta T_{bi}}\frac{1}{T_{pi}K_i}\right]x_i$$
$$+\left[0,\ 0,\ 0,\ \frac{\partial\Delta P_{Ai}}{\partial\Delta T_{fi}}\frac{1}{T_{pi}K_i}\right]x_{i+1}+\left[\frac{1}{T_{si}},\ 0\right]u_i$$
$$+\left[\frac{\partial\Delta P_{Ai}}{\partial\Delta H_i}\frac{1}{T_{pi}K_i},\ \frac{1}{T_{si}}\right]w_i+[0,\ 1]\frac{dw_i}{dt} \quad \text{(A. 1)}$$

$$\frac{dx_{i,2}}{dt}=\left[\frac{\partial\Delta P_{Ai}}{\partial\Delta h_i}\frac{1}{T_{pi}},\ -\frac{1}{T_{pi}},\ 0,\ \frac{\partial\Delta P_{Ai}}{\partial\Delta T_{bi}}\frac{1}{T_{pi}}\right]x_i$$
$$+\left[0,\ 0,\ 0,\ \frac{\partial\Delta P_{Ai}}{\partial\Delta T_{fi}}\frac{1}{T_{pi}}\right]x_{i+1}+\left[\frac{\partial\Delta P_{Ai}}{\partial\Delta H_i}\frac{1}{T_{pi}},\ 0\right]w_i \quad \text{(A. 2)}$$

$$\frac{dx_{i,3}}{dt}=\left[0,\ 0,\ -\frac{1}{T_{vi}},\ 0\right]x_i+\left[0,\ \frac{1}{T_{vi}}\right]u_i \quad \text{(A. 3)}$$

$$\frac{dx_{i,4}}{dt}=-\frac{Eb_i}{L_i}\left[\frac{\partial\Delta f_{i-1}}{\partial\Delta h_i}h_iV_{Ri}+V_{oi},\ 0,\ 0,\ \frac{\partial\Delta f_{i-1}}{\partial\Delta T_{bi-1}}h_{i-1}V_{Ri-1}\right]x_{i-1}$$
$$+\frac{Eb_i}{L_i}\left[\left(\frac{\partial\Delta f_i}{\partial\Delta h_i}h_iV_{Ri}+V_{oi}\right),\ 0,\ -(1+f_{i-1})h_{i-1},\right.$$
$$\left.\left(\frac{\partial\Delta f_i}{\partial\Delta T_{bi}}h_iV_{Ri}-\frac{\partial\Delta f_{i-1}}{\partial\Delta T_{fi-1}}h_{i-1}\ V_{Ri+1}\right)\right]x_i$$
$$+\frac{Eb_i}{L_i}\left[0,\ 0,\ (1+f_i)h_i,\ \frac{\partial\Delta f_i}{\partial\Delta T_{fi}}h_iV_{Ri}\right]x_{i+1}$$
$$+\frac{Eb_i}{L_i}\left[\frac{\partial\Delta f_i}{\partial\Delta H_i}h_iV_{Ri},\ 0\right]w_i-\frac{Eb_i}{L_i}\left[\frac{\partial\Delta f_{i-1}}{\partial\Delta H_{i-1}}h_{i-1}V_{Ri-1},\ 0\right]w_{i-1} \quad \text{(A. 4)}$$